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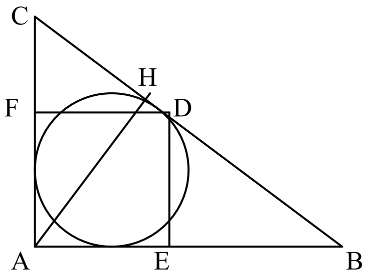
Dimitar Mitkov Kunchev
student, 15 years old
BABA TONKA High School of Mathematics
18 Ivan Vazov St.
ROUSSE 7000, BULGARIA

BRUCE SHAWYER
Department of Mathematics and Statistics
Memorial University of Newfoundland
St. John's, Newfoundland
CANADA. A1C 5S7

Problem 2440. Given: triangle *ABC* with $\angle BAC = 90^0$. The incircle of triangle *ABC* touches *BC* at *D*. Let *E* and *F* be the feet of the perpendiculars from *D* to *AB* and *AC* respectively. Let *H* be the foot of the perpendicular from *A* to *BC*. Prove that the area of the rectangle *AEDF* is equal to $\frac{AH^2}{2}$.

Solution 2440. Let *AB* = *c*; *AC* = *b*; *BC* = *a*. Then



$$CD = \frac{a+b-c}{2} \text{ and } DB = \frac{a+c-b}{2}.$$

$$\Delta EBD \sim \Delta ABC \Rightarrow \frac{ED}{AC} = \frac{BD}{BC} \Rightarrow ED = \frac{\frac{a+c-b}{2}}{a}.b$$

$$\Delta FDC \sim \Delta ABC \Rightarrow \frac{FD}{AB} = \frac{CD}{CB} \Rightarrow FD = \frac{\frac{a+b-c}{2}}{a}.c$$

We denote by *S* the area of the rectangle *AEDF*, so

$$S = ED.FD = \frac{1}{4a^2}(a+c-b)(a+b-c).b.c = \frac{a^2-(c-b)^2}{4a^2}.b.c =$$

$$= \frac{a^2-c^2+2bc-b^2}{4a^2}.b.c = \frac{2b^2c^2}{4a^2} = \frac{1}{2}\left(\frac{b.c}{a}\right)^2 = \frac{AH^2}{2}$$

We used Pythagora`s theorem for ΔABC and the formula for the altitude in right triangle: $AH = \frac{b.c}{a}$.