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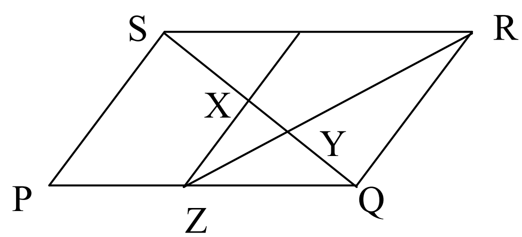
PROBLEM 2359. Let **PQRS** be a parallelogram. Let **Z** divide **PQ** internally in the ratio **k:l** . The line through **Z** parallel to **PS** meets diagonal **SQ** at **X**. The line **ZR** meets **SQ** at **Y**. Find the ratio **XY:SQ**.

SOLUTION.



Since $XZ \parallel RQ$ it follows that ΔXYZ is similar to ΔQYR and then $\frac{XY}{YQ} = \frac{XZ}{RQ}$.

PQRS is parallelogram so $RQ = PS$ and we get $\frac{XY}{YQ} = \frac{XZ}{PS}$. By the similar triangles

ΔXZQ and ΔSPQ we get $\frac{XZ}{PS} = \frac{ZQ}{PQ} = \frac{l}{k+l}$. Therefore $\frac{XY}{YQ} = \frac{l}{k+l}$. (1)

It is given that $XZ \parallel PS$, hence $\frac{ZQ}{PZ} = \frac{l}{k} = \frac{XQ}{SX} = \frac{XY}{SX} + \frac{YQ}{SX} = \frac{XY}{YQ}\frac{YQ}{SX} + \frac{YQ}{SX}$.

Using (1) we get: $\frac{l}{k} = \frac{YQ}{SX}\left(\frac{l}{k+l} + 1\right) \Rightarrow \frac{YQ}{SX} = \frac{l(k+l)}{k(2l+k)}$ (2)

From $XZ \parallel PS$ we get that $\frac{SX}{SQ} = \frac{PZ}{PQ} = \frac{k}{k+l}$. (3)

Equations (1), (2) and (3) imply $\frac{XY}{SQ} = \frac{XY}{YQ}\frac{YQ}{SX}\frac{SX}{SQ} = \frac{l^2}{(k+l)(2l+k)}$.