Виж Crux Mathematicorum 7/2001, стр. 466.

2559. [200 : 305] *Proposed by Hojoo Lee, student, Kwangwoon University, Kangwon-Do, South Korea.*

Triangle ABC has incentre I. Show that $CA + AI = CB$ if and only if $\angle CAB = 2\angle ABC$.

Solution by Toshio Seimiya, Kawasaki, Japan; Mitko Kunchev, Baba Tonka School of Mathematics, Rousse, Bulgaria; and Gottfried Perz, Pestalozzigymnasium, Graz, Austria.

Since I is the incentre of ΔABC, we have $\angle ICA = \angle ICB$, $\angle IAC = \angle IAB$ and $\angle IBC = \angle IBA$. Hence $\angle CAI = 1/2\angle CAB$ and $\angle CBI = 1/2\angle ABC$. Let D be the point on AC, so that A is between C and D, and $AD = AI$. Then $\angle CDI = 1/2\angle CAI$ and therefore, $\angle CDI = 1/4\angle CAB$.

(1) Let $CA + AI = CB$. Then $CA + AI = CA + AD = CD$, so that $CD = CB$. Since $\angle DCI = \angle BCI$, the triangles CDI and CBI are congruent. Thus $\angle CDI = \angle CBI$. Hence $1/4\angle CAB = 1/2\angle ABC$. Therefore, $\angle CAB = 2\angle ABC$.

(2) Let $\angle CAB = 2\angle ABC$. Then $1/4\angle CAB = 1/2\angle ABC$, so that $\angle CDI = \angle CBI$. Since $\angle DCI = \angle CBI$, the triangles CDI and CBI are congruent, so that $CD = CB$. As $AD = AI$, we have $CD = CA + AD = CA + AI$. Therefore, $CA + AI = CB$.

From (1) and (2), it follows that $CA + AI = CB$ if and only if $\angle CAB = 2\angle ABC$.