Виж Crux Mathematicorum 6/1996, стр. 279.

2069. [1995: 235] *Proposed by D.J. Smeenk, Zaltbommel, the Netherlands.*

M is a variable point of side BC of ΔABC. A line through M intersects the lines AB in K and AC in L so that M is the mid-point of segment KL. Point K_1 is such that $ALKK_1$ is a parallelogram. Determine the locus of K_1 as M moves on segment BC.

II. Essentially the same s*olution by Tim Cross, King Edward's School,Birmigham, England; Hidetosi Fukagawa, Japan; Walther Janous, Ursulinengymnasium, Innsbruk, Austria; Mitko Christov Kunchev, Rousse, Bulgaria; Ashsih Kr. Singh, student, Kanpur, India.*

Take A to be the origin and set the vectors $\overrightarrow{AB} = \vec{B}$, etc. Then $\vec{M} = t\vec{B} + (1-t)\vec{C}$, where t varies from 0 to 1 as M moves from C to B. Suppose that $\vec{K} = k\vec{B}$ and that $\vec{L} = \lambda\vec{C}$. Because M is the mid-point of KL, we have

$$\frac{k\vec{B} + \lambda\vec{C}}{2} = t\vec{B} + (1-t)\vec{C},$$

so that $k = 2t$ and $\lambda = 2(1-t)$ (since $\vec{B}$ and $\vec{C}$ are linearly independent). Since $ALKK_1$ is a parallelogram, it follows that

$$\vec{K_1} = \vec{K} - \vec{L} = t\left(2\vec{B}\right) + (1-t)\left(-2\vec{C}\right).$$

Thus the locus of K_1 is the segment joining -2$\vec{C}$ (where $t = 0$) to 2$\vec{B}$ (where $t = 1$).